CERENKOV RADIATION FROM COLLISIONS OF STRAIGHT COSMIC (SUPER)STRINGS

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Abstract. We consider Cerenkov radiation which must arise when randomly oriented straight cosmic (super)strings move with relativistic velocities without intercommutation. String interactions via dilaton, two-form and gravity (gravity being the dominant force in the ultra-relativistic regime) leads to formation of superluminal sources which generate Cerenkov radiation of dilatons and axions. Though the effect is of the second order in the couplings of strings to these fields, its total efficiency is increased by high dependence of the radiation rate on the Lorentz-factor of the collision.

1. Introduction

Recently the early universe models involving strings and branes moving in higher-dimensional space-times received a renewed attention [1]-[4]. In particular, the problem of the dimensionality of space-time can be explored within the brane gas scenario [1]-[3]. Another new suggestion is the possibility of cosmic superstrings with lower tension than those in the field-theoretical GUT strings [3]. Superstrings as cosmic strings candidates revive the idea of the defect origin of cosmic structures and stimulate reconsideration of the cosmic string evolution with account for new features such as existence of the dilaton and antisymmetric form fields and extra dimensions. The main role in this evolution is played by radiation processes. The radiation mechanism which has been mostly studied in the past consists in formation of the excited closed loops which subsequently loose their excitation energy emitting gravitons [5] axions [6] and dilatons [7]-[10].

In this paper we consider the bremsstrahlung mechanism of string radiation [11] which works for initially unexcited strings undergoing a collision. We develop a classical perturbation scheme for two endless unexcited long strings which move one with respect to another in two parallel planes being inclined at an angle. It was shown earlier that in four space-time dimensions there is no gravitational bremsstrahlung under collision

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of straight strings [11]. This can be traced to absence of gravitons in 1+2 gravity. It is not a coincidence that in four dimensions there is no gravitational renormalization of the string tension either [13]. But there is no such dimensional argument in the case of the axion field there such dimensional argument and it was demonstrated that string bremsstrahlung takes place indeed [12] within the model in flat space. Here we extend this result to the full gravitating case including also the dilaton field. Strings interacts via the dilaton, axion and graviton exchange. Radiation arises in the second order approximation in the coupling constants provided the (projected) intersection point moves with superluminal velocity. Thus, the string bremsstrahlung can be viewed as manifestation of the Cherenkov effect.

2. String interactions

Consider a pair of relativistic strings $x^\mu = x^\mu_n(\sigma^a_n)$, $\mu = 0,1,2,3$, $\sigma_a = (\tau,\sigma)$, $a = 0,1$, where $n = 1,2$ is the index labelling the two strings. The 4-dimensional space-time metric signature $+,- - -$ and $(+,-)$ for the string world-sheets metric signature. Strings interact via the gravitational $g_{\mu\nu} \equiv \eta_{\mu\nu} + h_{\mu\nu}$, dilatonic $\phi(x)$ and axion (Kalb-Ramond) field $B_{\mu\nu}(x)$:

$$S = -\sum_n \int \left\{ \frac{\mu}{2}\partial_a x^\mu_n \partial_b x^\nu_n g_{\mu\nu}\gamma^{ab}\sqrt{-\gamma}\exp^{2\alpha_n\phi} + 2\pi f \partial_a x^\mu_n \partial_b x^\nu_n \epsilon^{ab} B_{\mu\nu}\right\} d^2\sigma$$
$$+ \int \left\{ 2\partial_\mu\phi\partial_\nu\phi g^{\mu\nu} + \frac{1}{6}H_{\mu\nu\rho}H^{\mu\nu\rho}\mathrm{e}^{-4\alpha\phi} - \frac{R}{16\pi G}\right\}\sqrt{-g}\, d^4x. \tag{1}$$

Here μ_n are the (bare) string tension parameters, α and f are the corresponding coupling parameters, $\epsilon^{01} = 1$, γ_{ab} is the induced metric on the world-sheets. In what follows, we linearize the dilaton exponent as $\mathrm{e}^{2\alpha\phi} \simeq 1 + 2\alpha\phi$.

The totally antisymmetric axion field strength is defined as $H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu}$. Variation of the action (1) over x^μ_n leads to the equations of motion for strings

$$\partial_a\left(\mu\partial_b x^\nu_n g_{\mu\nu}\gamma^{ab}\sqrt{-\gamma}\mathrm{e}^{2\alpha\phi} + 4\pi f \partial_b x^\nu_n \epsilon^{ab}B_{\mu\nu}\right)$$
$$-\mu\alpha\partial_a x^\alpha_n \partial_b x^\beta_n g_{\alpha\beta}\gamma^{ab}\sqrt{-\gamma}\mathrm{e}^{2\alpha\phi}\partial_\mu\phi - \frac{\mu}{2}\partial_a x^\alpha_n\partial_b x^\beta_n\gamma^{ab}\sqrt{-\gamma}\mathrm{e}^{2\alpha\phi}\partial_\mu g_{\alpha\beta} = 0. \tag{2}$$

Variation with respect to field variables ϕ, $B_{\mu\nu}$ and $g_{\mu\nu}$ leads to the dilaton equation:

$$\partial_\mu\left(g^{\mu\nu}\partial_\nu\phi\sqrt{-g}\right) + \frac{\alpha}{6}H^2\mathrm{e}^{-4\alpha\phi} + \frac{\mu\alpha}{4}\int \partial_a x^\mu_n \partial_b x^\mu_n g_{\mu\nu}\gamma^{ab}\mathrm{e}^{2\alpha\phi}\delta^4(x - x_n(\sigma_n))d^2\sigma = 0, \tag{3}$$

the axion equation:

$$\partial_\mu\left(H^{\mu\nu\lambda}\mathrm{e}^{-4\alpha\phi}\sqrt{-g}\right) + 2\pi f\int \partial_a x^\nu_n \partial_b x^\lambda_n \epsilon^{ab}\delta^4(x - x_n(\sigma_n))d^2\sigma = 0 \tag{4}$$

and the Einstein equations: $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(\overset{\phi}{T}_{\mu\nu} + \overset{B}{T}_{\mu\nu} + \overset{st}{T}_{\mu\nu})$,
$\overset{st}{T}_{\mu\nu} = \sum \mu \int \partial_a x_{\mu\, n}\partial_b x_{\nu\, n}\gamma^{ab}\sqrt{-\gamma}\mathrm{e}^{2\alpha\phi}\frac{\delta^4(x - x_n(\sigma_n))}{\sqrt{-g}}d^2\sigma$,
$\overset{\phi}{T}_{\mu\nu} = 4\left(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\nabla\phi)^2\right)$, $\overset{B}{T}_{\mu\nu} = \left(H_{\mu\alpha\beta}H^{\alpha\beta}_\nu - \frac{1}{6}H^2 g_{\mu\nu}\right)\mathrm{e}^{-4\alpha\phi}$.

Our calculation follows the approach of [11]-[12] and consists in constructing solutions of the string equations of motion and dilaton, axion and graviton iteratively using the coupling constants α, f, G as expansion parameters.

The total dilaton, axion and graviton fields are the sums due to contributions of two strings: $\phi = \phi_1 + \phi_2$, $B^{\mu\nu} = B_1^{\mu\nu} + B_2^{\mu\nu}$, $h^{\mu\nu} = h_1^{\mu\nu} + h_2^{\mu\nu}$. Since in the zero order the strings are moving freely , the first order dilaton $\overset{1}{\phi}_n$, axion $\overset{1}{B}{}_n^{\mu\nu}$ and graviton variables $\overset{1}{h}{}_n^{\mu\nu}$ do not contain radiative components. Substituting them into the Eq. (2) we then obtain the first order deformations of the world-sheets $\overset{1}{x}{}^{\mu}$, which are naturally split into contributions due to dilaton, axion and graviton exchange: $\overset{1}{x}{}_n^{\mu} = \overset{1}{x}{}^{\mu}_{n(\phi)} + \overset{1}{x}{}^{\mu}_{n(B)} + \overset{1}{x}{}^{\mu}_{n(h)}$.

Radiation arises in the second order field terms $\overset{2}{\phi}_n$ and $\overset{2}{B}{}_n^{\mu\nu}$ which are generated by the first order currents $\overset{1}{J}_{(\phi)}, \overset{1}{J}{}^{\mu\nu}_{(B)}$ in the dilaton and axion field equations ((3),(4)).

Note that gravitational radiation in four dimensions is absent [11], so we do not consider the second order graviton equation. The dilaton and axion radiation power can be computed as the reaction work given by the half sum of the retarded and advanced fields upon the sources [12]. The final formula for the dilaton and axion bremsstrahlung from the collision of two global strings can be obtained analytically in the case of the ultrarelativistic collision with the Lorentz factor $\gamma = (1 - v^2)^{-1/2} \gg 1$. We assume the BPS condition for the coupling constants [13] $\alpha\mu = 2\sqrt{2}\pi f$. The main contribution to radiation turns out to come from the graviton exchange terms. The spectrum has an infrared divergence due to the logarithmic dependence of the string interaction potential on distance, so a cutoff length Δ has to be introduced:

$$P^{(\phi)} = \frac{200}{3}\pi G^2\alpha^2\mu^4 L\kappa^5 (f(y) + \frac{1}{25} f_1(y)), \quad P^{(B)} = \frac{16\pi^3 G^2\mu^2 L f^2\kappa^5}{3}(f(y) - f_2(y)), \quad (5)$$

where L-length of the string, $y = \frac{d}{\gamma\kappa\Delta}$, $\kappa = \gamma\cos\alpha$, α is the strings inclination angle, d is the impact parameter and $f(y) = 12\sqrt{\frac{y}{\pi}}\, {}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-y\right) - 3\ln\left(4y\mathrm{e}^{C}\right)$, $f_1(y) = \mathrm{erfc}(\sqrt{y})\left(\frac{8}{3}y^3 - 30y^2 + 114y + \frac{169}{2}\right) - \frac{\mathrm{e}^{-y}\sqrt{y}}{\pi}\left(\frac{8}{3}y^2 - \frac{94}{3}y + 131\right)$, $f_2(y) = \mathrm{erfc}(\sqrt{y})\left(\frac{8}{3}y^3 + 6y^2 - 6y - \frac{5}{2}\right) - \frac{\mathrm{e}^{-y}\sqrt{y}}{\pi}\left(\frac{8}{3}y^2 + \frac{14}{3}y - 7\right)$, F is the generalized hypergeometric function and C is the Euler's constant.

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